THROUGHPUT PERFORMANCE OPTIMIZATION IN COGNITIVE RADIO NETWORKS BASED ON STOCHASTIC GEOMETRY

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ABSTRACT
We consider a cognitive radio (CR) network using the ALOHA MAC protocol. There are \( N \) wireless channels. The location of users in the network is modeled by a Poisson point process. We assume that the intensities of primary users (PUs) for channels are different. We focus on the throughput performance of an arbitrary secondary user (SU). Each SU selects a channel randomly and decides whether to be active or not with a common probability based on the channel selection. Then each SU senses the selected channel and decides whether the channel is idle or not based on the received signal to interference and noise ratio (SINR).

We derive a closed-form expression of the throughput and the optimal access probability (AP) that maximizes the throughput. These results give an insight on how to optimally design a CR network when SUs adopt the ALOHA MAC protocol.

POISSON POINT PROCESS

Let \( \Lambda \) be a locally finite non-null measure on \( \mathbb{R}^d \).

Definition 1 The Poisson point process \( \Phi \) of intensity measure \( \Lambda \) is defined by means of its finite dimensional distributions:

\[
\{ \Phi(A_1) = n_1, \ldots, \Phi(A_k) = n_k \} = \prod_{i=1}^{k} \left( e^{-\Lambda(A_i)} \frac{\Lambda(A_i)^{n_i}}{n_i!} \right),
\]

for every \( k = 1, 2, \ldots \) and all bounded, mutually disjoint sets \( A_i \) for \( i = 1, \ldots, k \). If \( \Lambda(dx) = \lambda dx \) is a multiple of Lebesgue measure in \( \mathbb{R}^d \), we call \( \Phi \) a homogeneous Poisson p.p. and \( \lambda \) is its intensity parameter.

Laplace Functional

Definition 2 The Laplace functional \( \mathcal{L} \) of a p.p. \( \Phi \) is defined by the following formula

\[
\mathcal{L}_\Phi(f) = \left[ e^{-\int_{\mathbb{R}^d} f(x) \Phi(dx)} \right],
\]

where \( f \) runs over the set of all non-negative functions on \( \mathbb{R}^d \).

Proposition 1 The Laplace functional of the Poisson p.p. of intensity measure \( \Lambda \) is

\[
\mathcal{L}_\Phi(f) = e^{-\int_{\mathbb{R}^d} (1-e^{-f(x)}) \Lambda(dx)}. \tag{1}
\]
Palm Theory

**Definition 3** Given a point process with a locally finite mean measure, the distribution $P_x^t(\cdot)$ is called the reduced Palm distribution of $\Phi$ given a point at $x$.

**Theorem 2 (Reduced Campbell-Little-Mecke Formula).** For all non-negative functions defined on $\mathbb{R}^d \times \mathbb{M}$

$$\left[ \int_{\mathbb{R}^d} f(x, \Phi - \varepsilon_x) \Phi(dx) \right] = \int_{\mathbb{R}^d} \int_{\mathbb{M}} f(x, \phi) P_x^t(d\phi) M(dx).$$

**Theorem 3 (Slivnyak-Mecke Theorem).** Let $\Phi$ be a Poisson p.p. with intensity measure $\Lambda$. For $\Lambda$ almost all $x \in \mathbb{R}^d$,

$$P_x(\cdot) = \{ \Phi \in \cdot \};$$

that is, the reduced Palm distribution of the Poisson p.p. is equal to its (original) distribution.

**MAIN RESULTS**

**Theorem 4** The throughput $T$ of an arbitrary SU is given by

$$T(a) = \sum_{k=1}^{N} \frac{1}{N} \alpha_k \mathcal{L}_{I_{P,k}}(\mu T \alpha) \mathcal{L}_{I_{P,k}}(\mu T R^\alpha) \mathcal{L}_{I_{S,k}}(\mu T R^\alpha)$$

where

$$\mathcal{L}_{I_{P,k}}(\mu T R^\alpha) = \exp \left\{ -2\pi \left( \frac{p_k}{p_k + q_k} \lambda_{p,k} \right) (TR^\alpha)^{2/\alpha} \frac{\pi}{\alpha \sin(2\pi/\alpha)} \right\}$$

and

$$\mathcal{L}_{I_{S,k}}(\mu T R^\alpha) = \exp \left\{ -2\pi \left( \frac{1}{N} \lambda_{s} \right) (TR^\alpha)^{2/\alpha} \frac{\pi}{\alpha \sin(2\pi/\alpha)} \right\} .$$

**Theorem 5** The optimal AP value $a_k^*$ that maximizes the throughput of an arbitrary SU is given by

$$a_k^* = \frac{1}{2\pi \frac{1}{N} \lambda_{s} (TR^\alpha)^{2/\alpha} \frac{\pi}{\alpha \sin(2\pi/\alpha)} \mathcal{L}_{I_{P,k}}(\mu T R^\alpha) \exp(-1)}.$$ 

**Theorem 6** The optimal throughput $T^*$ of an arbitrary SU in the CR network satisfies that

$$T^*(a^*) = \sum_{k=1}^{N} \frac{1}{2\pi \frac{1}{N} \lambda_{s} (TR^\alpha)^{2/\alpha} \frac{\pi}{\alpha \sin(2\pi/\alpha)} \mathcal{L}_{I_{P,k}}(\mu T R^\alpha) \exp(-1)} .$$

**REFERENCES**

