Method of moving frames to solve the time-dependent Maxwell’s equations on anisotropic curved surfaces

Ehsan Kazemi and Sehun Chun

1) Underwood International College, Yonsei University, South Korea

Corresponding Author: Ehsan Kazemi, ehsan.kazemy@yonsei.ac.kr

ABSTRACT

A high-order accurate method of moving frames for the solution of linear conservation laws in geometrically complex domain has been recently developed in [2]. As our main example we include a detailed development and analysis of a scheme for the solution of Maxwell’s equation in a three-dimensional domain. The fully unstructured spatial discretization is made possible by the use of high-order nodal basis, while the equation themselves are satisfied in a discontinuous Galerkin form by employing upwind flux with the boundary conditions being enforced weakly. This scheme solve Maxwell’s equations on curved surfaces without the metric tensor and composite meshes. Accuracy, stability, and convergence of the semidiscrete approximation to Maxwell’s equations is established rigorously and bounds on the growth of the global divergence error are provided. This sets the stage for the presentation of examples, verifying the theoretical results, and illustrating the versatility, flexibility and robustness when solving two- and three- dimensional benchmark problems in computational electromagnetics.

MOVING FRAMES FOR MAXWELL’S EQUATIONS

Consider the Maxwell’s equations such as

\[
\hat{\mu} \frac{\partial H}{\partial t} = - \nabla \times E,
\]

\[
\hat{\varepsilon} \frac{\partial E}{\partial t} = \nabla \times H.
\]

Moving frames form Maxwell’s equations are constructed in the similar manner as those for other equations [3,4]. Consider a curved element \(M^e\) of a sufficiently smooth surface \(M\) such that \(M = \{ \bigcup_i M^i | M^i \cap M^j = \delta^i_j \}\). Suppose that \(M^e\) is sufficiently smooth to satisfy that it is manifold and locally Euclidean. For every grid point in \(M^e\), construct three orthogonal unit vectors. Let us denote the constructed moving frames by \(e^i\) for \(1 \leq i \leq 3\).

Suppose by manipulating \(e^i\) that \(\varepsilon^i = \mu^i = 1\) for \(1 \leq i \leq 3\). Following [1], the DG formulation of the MMF-Maxwell scheme can be written as, for each \(i\), \(1 \leq i \leq 3\),

\[
\int \frac{\partial H^i}{\partial t} \varphi dx + \int_{\partial M} e^i \cdot (n \times E^e) \varphi ds
\]

\[
+ \sum_{m=1, m \neq i}^3 \left[ - \int E^m \nabla \varphi \cdot e^{mi} dx + \int E^m e^m \cdot (\nabla \times e^i) \varphi dx \right] = 0,
\]

where \(e^i\) are the constructed moving frames.
\[
\int \frac{\partial E_i^i}{\partial t} \varphi dx - \int_{\partial \mathcal{M}} e_i^i \cdot (n \times H^*) \varphi ds \\
+ \sum_{m=1, m \neq i}^3 \left[ \int H^m \nabla \varphi \cdot e_{mi}^i dx - \int H^m e_i^m \cdot (\nabla \times e_i^i) \varphi dx \right] = 0,
\]

where the flux terms are expressed as

\[
-e_i^i \cdot (n \times E^*) = e_i^i \cdot \frac{-\sum_{m=1, m \neq i}^3 n \times \{\{Y_{mi}E\} + 0.5n \times [\mathbf{H}]\}}{\sum_{m=1, m \neq i}^3 \{\{Y_{mi}\}\}},
\]

\[
e_i^i \cdot (n \times H^*) = e_i^i \cdot \frac{\sum_{m=1, m \neq i}^3 n \times \{\{Z_{mi}H\} + 0.5n \times [\mathbf{E}]\}}{\sum_{m=1, m \neq i}^3 \{\{Z_{mi}\}\}},
\]

where we let

\[
Z_{mi}^\pm = (Y_{mi})^{-1} = 1, \quad 1 \leq i, m \leq 3
\]

REFERENCES


