Numerical Osher method for Euler equations with nonlinear equations of state

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ABSTRACT

For the numerical simulation of detonation of condensed phase explosives, a complex equation of state (EOS), such as the Jones-Wilkins-Lee (JWL) EOS or the Cochran-Chan (C-C) EOS, are widely used. However, when a conservative scheme is used for solving the Euler equations with such equations of state, a spurious solution across the contact discontinuity, a well known phenomenon in multi-fluid systems, arises even for single materials. In this work, we develop a generalised Osher-type scheme in an adaptive primitive-conservative framework to overcome the aforementioned difficulties. Resulting numerical solutions are compared with the exact solutions and with the numerical solutions from the Godunov method in conjunction with the exact Riemann solver for the Euler equations with Mie-Grüneisen form of equations of state, such as the JWL and the C-C equations of state. The adaptive scheme is extended to second order and its empirical convergence rates are presented, verifying second order accuracy for smooth solutions. Through a suite of several tests problems in one and two space dimensions we illustrate the failure of conservative schemes and the capability of the proposed methods to overcome the difficulties.

INTRODUCTION

When the Euler equations for single material are solved with a sufficiently nonlinear equation of state, the numerical solution of a conservative scheme shows spurious oscillation in pressure and velocity [1,2]. Although the failure of conventional numerical methods combined with complex equations of state is of practical concern, this issue has not yet been widely investigated; there have been only a few attempts [1,3] to cure the failure when solving the Euler equations with complex equations of state.

In this work, as a simple and straightforward cure for avoiding spurious solutions, we adopt an adaptive primitive-conservative framework [4] whereby a conservative scheme is used at shock waves only and a primitive scheme is used elsewhere. Within this framework we choose to further develop the generalised Osher-type Riemann solver presented in [5], called here the Dumbser-Osher-Toro Riemann solver (or DOT solver). This scheme has some general attractive features, such as robustness for low density flows, entropy satisfaction, good behaviour for slowly-moving shocks and smoothness. It also has attractive aspects for the present study, namely: 1) The implementation of general equations of state is quite simple as the solver re-
mains independent of any equation of state by only requiring the speed of sound for the evaluation of the eigenstructure of the relevant system. 2) As both primitive and conservative versions of the DOT solver share the same eigenstructure, the implementation of the primitive scheme is just a matter of excluding the transformation matrix, keeping the rest of the solver unchanged.

**NUMERICAL RESULT**

Figure 1 shows the numerical solutions and the exact solution at $t = 40 \mu s$ for the advection of a density discontinuity in the flow of uniform velocity and pressure, with the C-C EOS using material parameters for Nitromethane. There arises non-physical pressure and velocity across the contact discontinuity in the numerical solutions of the conservative schemes, both the Godunov method and the conservative DOT method, although the discontinuity is within a single material. By using the adaptive version of DOT scheme, practically only the primitive version is activated for this problem as there is no shock wave involved in the flow, and the spurious oscillations are cleared in the numerical solution.

![Figure 1](image_url)

**REFERENCES**