EFFICIENT EMBEDDED FORMULA FOR CHEBYSHEV
COLLOCATION METHODS

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ABSTRACT

In this talk, we are concerned with embedded formulae of the Chebyshev collocation methods [1] developed recently. We introduce two Chebyshev collocation methods based on generalized Chebyshev interpolation polynomials [2], which are used to make an automatic integration method. We apply an elegant algorithm of generalized Chebyshev interpolation increasing the node points to make an error estimate for the lower order solution. Especially, we show that the algorithm enable us to make embedded formula, which allows the usage of larger time step sizes and the total computational costs required in simplified Newton-iteration can be reduced.

PRIMARY TARGET PROBLEM

The target problem we are concerned with is an initial value problem (IVP) of the form

\[ \phi'(t) = f(t, \phi(t)), \quad t \in (t_0, t_f]; \quad \phi(t_0) = \phi_0, \]

where \( \phi(t) := [\phi_1(t), \cdots , \phi_d(t)]^T \) and \( f(t, \phi(t)) := [f_1(t, \phi(t)), \cdots , f_d(t, \phi(t))]^T \) are assumed to be sufficient smooth for the simplicity of the analysis in the talk. This talk particularly focuses on the development of embedded formula for the collocation method, which is one of several widespread implicit Runge-Kutta (RK) schemes and uniquely defined by the choice of collocation points [3,4].

Main Contribution and Conclusions

The main contribution is to achieve an efficient embedded formula for the collocation method, using the idea for increasing the number of the zeros of the Chebyshev polynomials. More precisely, the collocation points are taken as the zeros of the generalized Chebyshev polynomials, consisting the Chebyshev-Gauss-Lobatto (CGL) points \( \{\eta_j\}_{j=0}^4 \) together with two extra points, \( \{\nu_k\}_{k=0}^1 \), given by

\[ \nu_k := \frac{1}{2} \left( \cos \left( \frac{k}{4} + \frac{3}{8} \pi \right) + 1 \right), \quad k = 0, 1, \]

where \( \eta_j \) and \( \nu_k \) are the zeros of the Chebyshev polynomials \( T_6^*(s) - T_3^*(s) \) and \( T_2^*(s) - \cos\left(\frac{3\pi}{4}\right) \), respectively [3, 4]. Here \( T_k^*(s) \) denotes the shifted Chebyshev polynomial of the first kind of
degree of k. An embedded formula of the Chebyshev collocation method for stiff problems has been constructed and analyzed. Instead of the traditional method to estimate the local truncation error [5], we developed a new embedded formula to calculate the lower order solution. To achieve this, the eigenvalues of the complex linear systems, derived from the collocation method for the lower order solution, are replaced by those of the higher order solution. The developed embedded scheme yields to a wide stability region for the Dahlquists problem. The local truncation error, err(z), of the scheme goes to 0, as z goes to , which is similar to the asymptotic behavior of another widely used method, Radau5 [5, 6]. The convergence and stability analysis shows that the scheme has 8th order of convergence and A-stability. By using several numerical experiments, we have shown that the scheme has a higher order of convergence, better stability and lower computational costs, compared to existing methods.

REFERENCES