An improvement of convergence order of WENO method on exponential functions for Hamilton-Jacobi equations

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ABSTRACT

This study introduces a novel weighted essentially non-oscillatory (WENO) conservative finite difference scheme based on exponential polynomials for solving Hamilton–Jacobi equations. The main ingredient of this paper is to employ exponential polynomials to compute numerical Hamiltonian and new high-order smoothness indicators. To reconstruct the Hamiltonian, we use an interpolation method based on exponential polynomials constructed by Ha et al.[3], which interpolates sharp gradients and highly oscillatory features more efficiently than algebraic polynomial-based methods. The new smoothness indicators are developed using generalized undivided difference operators based on exponential vanishing moment properties. The proposed smoothness indicators rapidly decay to zero near singular areas so they can distinguish singularity from smooth regions more robustly than other indicators. As a result, the new WENO scheme attains the enhanced order of accuracy in smooth areas, even near the high-order critical points, while maintaining essential non-oscillatory behavior near singular areas. A collection of numerical experiments is presented with comparisons to demonstrate the shock-capturing ability of the proposed scheme.

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