DB-IFEM for multiphase flows in heterogeneous porous media with discontinuous capillary pressure

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ABSTRACT

The governing equations of multiphase flows in porous media is an important problem in various disciplines especially in petroleum engineering. However, when there some interfaces on domain, there are difficulties to solve the problems, even numerically. Firstly, when the material properties vary across some interface, the coefficients (permeability, capillary pressure, porosity, etc.) of the underlying p.d.e become discontinuous. Secondly, one has to consider the nonhomogeneous jump conditions along the interface.

When the capillary pressure become discontinuous, the saturation and the pressure variables become discontinuous. The jump conditions are studied in [3]. Until now, some DG methods give accurate and stable approximation for the problems since the exact jumps can be handled in bilinear form in terms of penalty [4].

So far, most of the numerical methods for multiphase flows problems in heterogeneous problems are based on fitted grids, which yield complex data structure. In this work, we will give numerical methods in some other direction based on immersed finite element methods (IFEM). The advantage of IFEM that it can be implemented on uniform grids. Hence the system can be solved efficiently (for example by multigrid solver [5]). We use immersed finite element method (IFEM) frameworks for the problems. In [1], discontinuous-bubble IFEM was developed for the elliptic interface problems with jumps. The idea is to use bubble functions with small support to handle the jumps. We can obtain homogeneous problems after subtracting bubble functions. By this way we can handle the discontinuity.

Let us describe our scheme. Once we apply DB-IFEM to approximate pressure variables, we compute (locally) Darcy velocity using the frameworks of mixed finite volume [2]. After that saturation variable is solved by Euler backward scheme. We use control volume with up-winding scheme for the saturation variable. As far as the author' knowledge concerned, this kind of scheme is developed for the first time.

We present numerical experiments. We observe that the methods give plausible solutions.

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