A parallel asymmetric particle Gaussian mixture filter for state-space estimation of highly nonlinear oscillators

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ABSTRACT

Estimating internal system states from a series of measurements is essential for understanding biological oscillators. However, this task is challenging because such systems often exhibit both slow and fast manifolds, resulting in highly nonlinear dynamics. This undermines a key assumption of the widely used Kalman–Bucy filter—that system dynamics can be locally approximated by a single symmetric Gaussian distribution—and it also makes Monte Carlo sampling in particle filtering computationally expensive. Here, we develop the asymmetric particle Gaussian mixture filter (AP-GMF) that accurately estimates the system states in highly nonlinear oscillators. The AP-GMF adaptively splits and merges Gaussian particles by assessing the local degree of nonlinearity and the Kullback–Leibler divergence, respectively. This split–merge scheme approximates the asymmetric state distributions more accurately than the recently developed asymmetric particle population density method, while requiring fewer particles. Furthermore, the AP-GMF consistently outperforms existing filters, such as the level-set Kalman filter and the continuous–discrete cubature Kalman filter, on the van der Pol oscillator, a widely used model of highly nonlinear oscillatory dynamics, across a range of parameters. Lastly, we provide a parallel AP-GMF implementation, enabling high-fidelity state estimation with a reasonable computational cost.

REFERENCES

References are to be listed at the end of the paper in the order of the reference, and are referred to in the paper by the numbers in brackets such as [1, 2]. Style the reference list according to the following examples.

1. A. L. Hodgkin, A. F. Huxley, B. Katz, Measurement of current-voltage relations in the membrane of the giant axon of loligo, The Journal of physiology 116 (4) (1952) 424.
2. B. Van Der Pol, J. Van Der Mark, Lxxii. the heartbeat considered as a relaxation oscillation, and an electrical model of the heart, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 6 (38) (1928) 763–775.
3. M. E. Jewett, R. E. Kronauer, Refinement of limit cycle oscillator model of the effects of

light on the human circadian pacemaker, Journal of Theoretical Biology 192 (4) (1998) 455–465.
4. D. B. Forger, Biological clocks, rhythms, and oscillations: the theory of biological timekeeping (2024).
5. D. W. Kim, C. Chang, X. Chen, A. C. Doran, F. Gaudreault, T. Wager,G. J. DeMarco, J. K. Kim, Systems approach reveals photosensitivity and per 2 level as determinants of clock-modulator efficacy, Molecular systems biology 15 (7) (2019) e8838.
6. J. E. Stone, S. Postnova, T. L. Sletten, S. M. Rajaratnam, A. J. Phillips, Computational approaches for individual circadian phase prediction in field settings, Current Opinion in Systems Biology 22 (2020) 39–51.
7. D.-J. Dijk, J. F. Duffy, Novel approaches for assessing circadian rhyth- micity in humans: a review, Journal of biological rhythms 35 (5) (2020) 421–438.
8. K. J. Reid, Assessment of circadian rhythms, Neurologic clinics 37 (3) (2019) 505.
9. S. S¨arkk¨a, L. Svensson, Bayesian filtering and smoothing, Vol. 17, Cam- bridge university press, 2023.
10. I. Arasaratnam, S. Haykin, Cubature kalman filters, IEEE Transactions on automatic control 54 (6) (2009) 1254–1269.
11. I. Arasaratnam, S. Haykin, T. R. Hurd, Cubature kalman filtering for continuous-discrete systems: theory and simulations, IEEE Transactions on Signal Processing 58 (10) (2010) 4977–4993.
12. N. J. Gordon, D. J. Salmond, A. F. Smith, Novel approach to nonlinear/non-gaussian bayesian state estimation, in: IEE proceedings F (radar and signal processing), Vol. 140, IET, 1993, pp. 107–113.
13. F. Daum, J. Huang, Curse of dimensionality and particle filters, in: 2003 IEEE aerospace conference proceedings (Cat. No. 03TH8652), Vol. 4, IEEE, 2003, pp. 4 1979–4 1993.
14. C. Snyder, T. Bengtsson, P. Bickel, J. Anderson, Obstacles to high- dimensional particle filtering, Monthly Weather Review 136 (12) (2008) 4629–4640.
15. G. Terejanu, P. Singla, T. Singh, P. D. Scott, A novel gaussian sum filter method for accurate solution to the nonlinear filtering problem, in: 2008 11th International Conference on Information Fusion, IEEE, 2008, pp.1–8.
16. D. W. Kim, M. P. Lee, D. B. Forger, Wearable data assimilation to estimate the circadian phase, SIAM Journal on Applied Mathematics 84 (3) (2023) S452–S475.
17. H. W. Sorenson, D. L. Alspach, Recursive bayesian estimation using gaussian sums, Automatica 7 (4) (1971) 465–479.
18. W. Shao, Z. Ge, Z. Song, Semisupervised bayesian gaussian mixture models for non gaussian soft sensor, IEEE Transactions on Cybernetics 51 (7) (2019) 3455–3468.
19. B. Zhang, Y. C. Shin, A gaussian mixture filter with adaptive refinement for nonlinear state estimation, Signal Processing 201 (2022) 108677.
20. F. Faubel, J. McDonough, D. Klakow, The split and merge unscented gaussian mixture filter, IEEE Signal Processing Letters 16 (9) (2009) 786–789.
21. P. H. Leong, S. Arulampalam, T. A. Lamahewa, T. D. Abhayapala, A gaussian-sum based cubature kalman filter for bearings-only tracking, IEEE Transactions on Aerospace and Electronic Systems 49 (2) (2013) 1161–1176.
22. K. Tuggle, R. Zanetti, Automated splitting gaussian mixture nonlinear measurement update, Journal of Guidance, Control, and Dynamics 41 (3) (2018) 725–734.
23. D. Alspach, H. Sorenson, Nonlinear bayesian estimation using gaussian sum approximations, IEEE transactions on automatic control 17 (4) (2003) 439–448.
24. K. Ito, K. Xiong, Gaussian filters for nonlinear filtering problems, IEEE transactions on automatic control 45 (5) (2002) 910–927.
25. R. Chen, J. S. Liu, Mixture kalman filters, Journal of the Royal Statistical Society: Series B (Statistical Methodology) 62 (3) (2000) 493–508.
26. G. Terejanu, P. Singla, T. Singh, P. D. Scott, Uncertainty propagation for nonlinear

dynamic systems using gaussian mixture models, Journal of guidance, control, and dynamics 31 (6) (2008) 1623–1633.
27. P. Singla, T. Singh, A gaussian function network for uncertainty propagation through nonlinear dynamical system, in: 18th Annual Space Flight Mechanics Meeting, 2008, pp. 851–864.
28. N. Wang, D. B. Forger, The asymmetric particle population density method for simulation of coupled noisy oscillators, Journal of Computational Physics 488 (2023) 112157.
29. A. R. Runnalls, Kullback-leibler approach to gaussian mixture reduction, IEEE Transactions on Aerospace and Electronic Systems 43 (3) (2007) 989–999.
30. Y. Wang, H. Zhang, Accurate gaussian sum-filter for continuous-discrete nonlinear systems with non-gaussian noise, in: 2018 10th International Conference on Communications, Circuits and Systems (ICCCAS), IEEE, 2018, pp. 140–145.
31. N. Wang, D. Forger, The level set kalman filter for state estimation of continuous-discrete systems, IEEE Transactions on Signal Processing (2021).
32. A. G. Wills, J. Hendriks, C. Renton, B. Ninness, A bayesian filtering algorithm for gaussian mixture models (2023). arXiv:1705.05495. URL https://arxiv.org/abs/1705.05495