GLOBAL SENSITIVITY ANALYSIS FOR MULTIVARIATE OUTPUTS USING POLYNOMIAL CHAOS-BASED SURROGATE MODEL

Xiang SUN [1] , Yun Young CHOI [1] and Jung-Il CHOI [1]

1) *Department of Computational Science and Engineering, Yonsei University, Seoul 03722, KOREA*

We propose an efficient global sensitivity analysis method for multivariate outputs that applies polynomial chaos-based surrogate models to vector projection-based sensitivity indices. These projection-based sensitivity indices, which are powerful measures of the comprehensive effects of uncertain model inputs on multiple outputs, are conventionally estimated by the Monte Carlo simulations that incur prohibitive computational costs for many practical problems. Here, the projection-based sensitivity indices are efficiently estimated via two polynomial chaos-based surrogates: sparse polynomial chaos expansion and a proper orthogonal decomposition-based polynomial chaos expansion. Compared to the Monte Carlo simulations, the sparse polynomial chaos expansion usually approximates the original model with only a few number of samples. Furthermore, there is no more extra cost on the estimation of the sensitivity indices because the sensitivity indices are a function of polynomial chaos expansion coefficients. However, this approach can be computationally intensive if the number of the model outputs is large and, in some cases, can seriously reduce the applicability of this approach to realistic problems. To address this, we developed a new polynomial chaos expansion based on proper orthogonal decomposition, which can reduce the dimension of the model output and can estimate the statistical quantities as easily as the general polynomial chaos expansion. The new method reduces the computational cost by projecting the model output into a low-dimension space expanded by its principle components using proper orthogonal decomposition, and approximating the projection coefficients using a polynomial chaos expansion. Several numerical examples with various types of outputs are tested to validate the proposed methods; the results demonstrate that the polynomial chaos-based surrogates are more efficient than Monte Carlo simulations at estimating the sensitivity indices, even for models with a large number of outputs. Furthermore, for models with only a few outputs, polynomial chaos expansion alone is preferable, whereas for models with a large number of outputs, implementation with proper orthogonal decomposition is the best approach. Additionally, the results indicate that the estimated vector projection based sensitivity indices can efficiently and reasonably quantify the input uncertainty contributions to the whole output domain. This implies that the present global sensitivity analysis can potentially be used to design practical experiments, or to improve experimental design or mathematical models in engineering problems.