Euler-Poisson system and its application to plasma physics

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ABSTRACT

In this talk, I will present a mathematical modelling of the plasma sheath using the Euler-Poisson system arising from plasma physics such as semiconductor industry. I will derive a hyperbolic type interface system using axiomatic approach and show the local existence of approximate interface system. This is a joint work with Mikhail Feldman and Marshall Slemrod [5].

DESCRIPTION OF THE PROBLEM

Consider a plasma consisting of cold ions and hot electrons confined to a domain \( \Omega = 3 - \Omega_0 \) which is exterior to a target \( \Omega_0 \subset 3 \). Both ions and electrons have constant temperature, the temperature of the ions being absolute zero Kelvin. The density of ions is denoted by \( n \), the density of electrons is \( e^{-\phi} \) (Boltzmann relation), \( -\phi \) is the potential field and \( u \) is the velocity of the ions. In this case, (E-P) reads

\[
\begin{align*}
\frac{\partial n}{\partial t} + \nabla \cdot (nu) &= 0, \quad (x, t) \in \Omega \times (0, \infty), \\
\frac{\partial u}{\partial t} + (u \cdot \nabla) u &= \nabla \phi, \\
\varepsilon^2 \Delta \phi &= n - e^{-\phi},
\end{align*}
\]

subject to initial and boundary conditions:

\[
(n, \phi)(x, 0) = (n_0, \phi_0)(x), \quad x \in \Omega,
\]

\[
\nabla \phi \cdot \nu_0 = \frac{g(t)}{\varepsilon}, \quad (n, t) \in \partial \Omega_0 \times [0, \infty).
\]

Here \( \varepsilon \) is proportional to the Debye length \( \lambda_D \) and \( \nu_0 \) is the exterior normal at the target boundary \( \partial \Omega_0 \). Typically away from the boundary \( \partial \Omega_0 \), the formal \( \varepsilon \to 0 \) limit in (E-P) can be used to yield the quasi-neutral relation \( n = e^{-\phi} \). However near the boundary \( \partial \Omega_0 \), this quasi-neutrality breaks down (see Section 2) and a sheath boundary layer of width \( \varepsilon \) forms.

In [7], Ha and Slemrod gave a description of sheath dynamics for the case of planar, cylindrically and spherically symmetric motion, generalizing earlier work of Daube and Riemann [8]. In this talk, we make no restriction as to symmetry and formulate the dynamics of the sheath interface in terms of a geometric level-set, where the dynamics of the sheath interface is based on a step-sheath model. In the step sheath model, the spatial-time domain is separated by a propagating sheath interface into distinct quasi-neutral and sheath regions. Particularly interesting in our approach is a set of equations describing the evolution of the sheath interface as a curvature driven flow. Specifically, we show that the sheath interface evolution is governed by the equations:
\[ \frac{\delta \psi}{\delta t} = 0, \quad \frac{\delta n}{\delta t} = n \nabla \cdot \nu, \quad (V + 1) + \frac{\nu}{n} = -\frac{1}{n} \nabla_s \cdot (V \nabla \ln n), \]

where

(i) the level set \( \mathcal{S}(t) = \{ (\cdot, t) : \psi(\cdot, t) = 0 \} \) is the sheath interface;
(ii) \( \frac{\delta}{\delta t} = \partial_t + V \nu \cdot \nabla \) is the normal time derivative following \( \mathcal{S}(t) \) and \( \nabla_s \) is the surface gradient on \( \mathcal{S}(t) \);
(iii) \( \nu \) is the exterior unit normal to \( \mathcal{S}(t) \). Since \( \nabla \cdot \nu \) is twice the mean curvature of \( \mathcal{S}(t) \), motion is curvature driven;
(iv) \( \psi \) is the ion current and \( n \) is the ion density on the sheath interface.

Usefulness of such models is seen in studying material processing and in particular the plasma source ion implantation (PSII) technique invented by Conrad and his collaborators. Other applications may be found in the related problems for the modelling of electron beam where again loss of quasi-neutrality is a crucial issue.

We note that in this talk we have taken the normal component of the electric field to be prescribed on the boundary \( \partial \Omega_0 \). This boundary condition was used by Cipolla and Silevitch in their study of plasma-sheath evolution and considerably simplifies the proof of the existence and uniqueness theorems presented in Section 7. On the other hand the derivation of the evolution equations for the sheath interface is independent of the boundary conditions. (An existence and uniqueness theorem for the boundary conditions used in \([7]\) = \( w, \phi = \phi_w \) on \( \partial \Omega_0 \) remains an open problem).

We list main assumptions (M) employed in this paper.

- (M1) The sheath interface is non-characteristic for the exterior quasi-neutral system (??) and whose (signed) normal speed satisfies
  \[ V \neq 0, -1. \]
- (M2) Continuity relation: \( n, \phi, \nabla n, \nabla, \nabla \phi \) and \( h \) are continuous across the sheath interface.
- (M3) Continuity of surface Laplacian of \( \phi \), i.e., \( \Delta_s \phi \) across the interface.
- (M4) The current density decays to zero at \( \infty \), i.e., for each \( t \geq 0 \),
  \[ \lim_{|x| \to \infty} h_i(t) = 0. \]
- (M5) The target boundary is \( C^2 \)-regular, i.e., the boundary can be represented by the graph of a \( C^2 \)-function locally.

REFERENCES


