ADAPTIVE CRANK-NICOLSON METHODS FOR
PARABOLIC PROBLEMS

Dongho KIM and Eun-Jae PARK
1) Department of University College, Yonsei University, Seoul 120-749, KOREA
2) Department of Mathematics, Yonsei University, Seoul 120-749, KOREA

ABSTRACT

In this paper we present a posteriori error estimators for the approximate solutions of linear
parabolic equations. We consider discretizations of the problem by discontinuous Galerkin
method in time corresponding to variant Crank-Nicolson schemes and continuous Galerkin
method in space. Especially, finite element spaces are permitted to change at different time
levels. Exploiting Crank-Nicolson reconstruction idea introduced by Akrivis, Makridakis &
Nochetto [1], we derive space-time a posteriori error estimators of second order in time for
variant Crank-Nicolson-Galerkin finite element methods.

INTRODUCTION

We consider the following linear parabolic equation

\[ u_t - \Delta u = f \quad \text{in } \Omega \times (0, T), \]
\[ u = 0 \quad \text{on } \partial \Omega \times (0, T), \]
\[ u(\cdot, 0) = u_0 \quad \text{in } \Omega, \]

where \( \Omega \subset \mathbb{R}^2 \) is a bounded convex polygonal domain; \( u = u(x, t) \), \( u_t \) denotes \( \partial u/\partial t \); \( f \in L^2(0, T; L^2(\Omega)) \); \( u_0 \in L^2(\Omega) \).

When we find a numerical approximate solution of partial differential equations we can
adaptively perform local grid refinement in subdomains with an a posteriori error estimator for
an efficient computation. An a posteriori error estimator is computed from the known values i.e.,
the given data of the problem and the computed numerical solutions. In this paper a posteriori
error estimates for the approximate solutions of linear parabolic problem are presented. We
consider discretizations of the problem by discontinuous Galerkin schemes corresponding to
modified Crank-Nicolson methods in time and continuous Galerkin finite element methods in
space (Crank-Nicolson-Galerkin schemes). Discontinuous Galerkin method used in this paper
is obtained from slight modifying of the standard continuous Galerkin method investigated by
Aziz & Monk [2]. This modification results from using the projection method in order to treat
approximate solutions obtained in varying spaces at different time levels.

A posteriori error estimators for the standard discontinuous Galerkin method have been
developed for parabolic problems in [5,6,3]. Especially, in [3] mixed finite element spaces
have been used for the space variable and the discontinuous Galerkin method corresponding
to modified backward Euler method in time. And for the \( \theta \)-scheme in time and conforming
finite elements in space have been for the heat equation in [8]. For backward Euler fully discrete
approximations to parabolic problems, a posteriori error estimates have been constructed by [7,4].

The purpose of this paper is to obtain optimal order estimators in the spaces \( L^\infty(0, T; L^2(\Omega)) \) for the scalar variables. That is, we recover a posteriori quantity of second order in time for the Crank-Nicolson-Galerkin fully discrete schemes on which it was open problem to get one of second order because of difficulty in obtaining optimal order convergence in time. But we successfully recover a posteriori quantity of second order in time for the Crank-Nicolson-Galerkin schemes by using the reconstruction idea introduced by Akrivis, Makridakis & Nochetto [1].

REFERENCES