Species extinction and permanence of an impulsively controlled two-prey one-predator system with seasonal effects

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ABSTRACT

The dynamical relationships between predator and prey can be represented by the functional response which refers to the change in the density of prey attached per unit time per predator as the prey density changes. One of well-known functional responses is the Beddington-DeAngelis functional response introduced by Beddington [3] and DeAngelis et al [5], independently. The main difference of this functional response from a classical Holling type ones is that this one contains an extra term presenting mutual interference by predators. There are a lot of factors to be considered in the environment to describe more realistic relationships between predators and prey. One of important factors is seasonality, which is a kind of periodic fluctuation varying with changing seasons. Also, the seasonality has an effect on various parameters in the ecological systems. For this reason, it is valuable to carry out research on systems with periodic ecological parameters which might be quite naturally exposed such as those due to seasonal effects of weather or food supply etc [4,18]. There are several ways to reflect the effects caused by the seasonality on ecological systems [12,13,23]. In this paper we consider the intrinsic growth rate $a$ of the prey population as periodically varying function of time due to seasonal variations. In other words we adopt $a_0 = a(1 + \epsilon \sin(\omega t))$ as the intrinsic growth rate of the prey. Here the parameter $\epsilon$ represents the degree of seasonality, $ae$ the magnitude of the perturbation in $a_0$ and $\omega$ the angular frequency of the fluctuation caused by seasonality.

Moreover, there are still some other factors that affect ecological system such as fire, flood, harvesting seasons etc, that are not suitable to be considered continually. These impulsive perturbations bring sudden change to the system. Such impulsive systems are found in almost every area of applied science and have been studied in many researches: impulsive birth [17,21], impulsive vaccination [6,19], chemotherapeutic treatment of disease [10,16]. In particular, the impulsively controlled prey-predator population systems have been investigated by a number of researchers [1,8,12–14,20,22,24–34]. Thus the field of research of impulsive differential equations with impulsive control terms seems to be a new growing interesting area in recent years. However, the two-prey and one-predator systems with seasonal effects and impulsive controls are less noticeable in spite of their importance. Now we develop the following new system with seasonality by bringing in a proportional periodic impulsive harvesting such as spraying pesticide for all
species and a constant periodic releasing for the predator at different fixed moment.

\[
\begin{align*}
    x_1'(t) &= x_1(t) \left( a_1 + \gamma_1 \sin(\theta_1 t) - b_1 x_1(t) - c_1 x_2(t) - \frac{\sigma_1 y(t)}{1 + d_1 x_1(t) + e_1 x_2(t) + \mu_1 y(t)} \right), \\
    x_2'(t) &= x_2(t) \left( a_2 + \gamma_2 \sin(\theta_2 t) - b_2 x_2(t) - c_2 x_1(t) - \frac{\sigma_2 y(t)}{1 + d_2 x_1(t) + e_2 x_2(t) + \mu_2 y(t)} \right), \\
    y'(t) &= y(t) \left( -a_3 + \frac{\sigma_3 x_1(t)}{1 + d_1 x_1(t) + e_1 x_2(t) + \mu_1 y(t)} + \frac{\sigma_4 x_2(t)}{1 + d_2 x_1(t) + e_2 x_2(t) + \mu_2 y(t)} \right), \\
    t &\neq nT, t \neq (n + \tau - 1)T, \\
    x_1(t^+) &= (1 - p_1)x_1(t), \\
    x_2(t^+) &= (1 - p_2)x_2(t), \\
    y(t^+) &= (1 - p_3)y(t), \\
    x_1(t^+) &= x_1(t), \\
    x_2(t^+) &= x_2(t), \\
    y(t^+) &= y(t) + q, \\
    \left( x_1(0^+), x_2(0^+), y(0^+) \right) &= (x_0, x_0, y_0),
\end{align*}
\]

where \( x_1(t), x_2(t) \) and \( y(t) \) represent the population density of two preys and the predator at time \( t \), respectively. The constant \( a_i (i = 1, 2) \) are called the intrinsic growth rates of the prey population, \( b_i (i = 1, 2) \) are the coefficients of intra-specific competition, \( c_i (i = 1, 2) \) are the parameters representing competitive effects between two preys, \( \sigma_i (i = 1, 2) \) are the per-capita rates of the predation of the predator, \( d_i (i = 1, 2) \) and \( e_i (i = 1, 2) \) are the half-saturation constants, the constant \( a_3 \) is the death rate of the predator, the terms \( \mu_i (i = 1, 2) \) scale the impact of the predator interference, \( \sigma_i (i = 3, 4) \) are the rates of the conversing prey into the predator, \( \lambda \) and \( \omega \) represent the magnitude and the frequency of the seasonal forcing terms, respectively, \( \tau \) and \( T \) are the period of spraying pesticides (harvesting) and the impulsive immigration or the stock of the predator, respectively, \( 0 \leq p_1, p_2, p_3 < 1 \) present the fraction of two preys and the predator which die due to harvesting or pesticides etc and \( q \) is the size of immigration or the stock of the predator.

The main purpose of this paper is to investigate the conditions for the extinction of the two preys and for the permanence of the system (1). In addition, we provide some numerical simulations to substantiate our theoretical results.

In this talk, we take into account the local and global asymptotical stabilities of prey-free periodic solutions of the systems (1) by using Floquet theory for the impulsive equation, small amplitude perturbation skills and comparison techniques, and next, prove that these systems are permanent under some conditions. Moreover, we establish the sufficient conditions under which one of the two prey is extinct and the remaining two species are permanent. Some numerical examples are given.

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