A Numerical Study for Absolute Conductivity Reconstruction: Projected Current Reconstruction and Harmonic $B_z$ Algorithms

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ABSTRACT
Magnetic resonance electrical impedance tomography (MREIT) is a conductivity imaging modality providing high-spatial resolution. During the injection of currents using several pairs of surface electrodes, we extract a component of the magnetic flux density, so called $B_z$, induced by the injected current using an MR scanner. Using the non-linear relation between $B_z$ data and conductivity, there have been numerous theories and experimental results for imaging reconstruction. In the stage of in-vivo animal or human experiments, conductivity value estimation or absolute conductivity reconstruction become one of important research issues for the diagnosis. In this talk, we discuss several issues for conductivity reconstruction for the conductivity value estimation and absolute conductivity imaging in MREIT.

INTRODUCTION
Last two decades, there have been remarkable developments of the conductivity imaging technology in magnetic resonance electrical impedance tomography (MREIT). Since the dedication of many researcher, MREIT conductivity imaging is now in the stage of in-vivo animal and human being imaging [1,2].

During the injection of currents using several pairs of surface electrodes $E_{\pm}$ to produce internal current density $J := (J_x, J_y, J_z)$, we measure a component of the magnetic flux density $B_z$ of $B := (B_x, B_y, B_z)$ induced by the internal current density $J$ using an MR scanner. Assume that the conductivity $\sigma$ is distributed in a three dimensional imaging object $\Omega \subset \mathbb{R}^3$. If we inject current $I$ through a pair of electrode $E_{\pm}$, the induced magnetic flux density $B_z$ satisfies the Biot-Savart’s Law:

$$B_z(r) = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\sigma((r'))[(x-x')\partial_y u(r') - (y-y')\partial_x u(r')]}{|r-r'|^3}dr' + \mathcal{H}(r)$$

where $\mu_0$ is the magnetic permeability of hydrogen, $r = (x, y, z)$ and $\mathcal{H}$ is a harmonic function in $\Omega$ due to exterior currents outside $\Omega$. The voltage potential $u$ in $\Omega$ satisfies the following
elliptic equation:
\[
\begin{align*}
\nabla \cdot (\sigma \nabla u) &= 0 \quad \text{in } \Omega \\
I &= \int_{E^+} \sigma \frac{\partial u}{\partial n} \, ds = -\int_{E^-} \sigma \frac{\partial u}{\partial n} \, ds \\
\nabla u \times n &= 0 \quad \text{on } E^+ \cup E^- \\
\sigma \frac{\partial u}{\partial n} &= 0 \quad \text{on } \partial \Omega \setminus (E^+ \cup E^-).
\end{align*}
\]

Here, \( n \) is the unit outward normal vector to \( \partial \Omega \). Note that the current density \( J = -\sigma \nabla u \).

In order to reconstruct the conductivity in MREIT, the harmonic \( B_z \) algorithm has been developed which is the first constructive algorithm with taking an advantage of using only one component of magnetic flux density \([3,4]\). Compared with previous studies \([5–8]\), the harmonic \( B_z \) algorithm has been regarded as a breakthrough toward the clinical modality of MREIT by eliminating the impractical object rotations in MR-scanner. After the invention of the harmonic \( B_z \) algorithm, there have been many successful results in MREIT, not only mathematical theory but pantom and animal experiments \([9–11]\).

Assume that \( B_{z,1} \) and \( B_{z,2} \) are corresponding to \( J_1 \) and \( J_2 \), respectively. By manipulating the Ampere’s law, we have the following identity about conductivity \( \sigma \), current density \( J \), and magnetic flux density \( B_z \):

\[
\nabla^2_{xy} \sigma = \frac{1}{\mu_0} \nabla_{xy} \cdot \begin{pmatrix}
-J_{y,1} & J_{x,1} \\
-J_{y,2} & J_{x,2}
\end{pmatrix}^{-1} \begin{pmatrix}
\nabla^2 B_{z,1} \\
\nabla^2 B_{z,2}
\end{pmatrix}.
\tag{2}
\]

The harmonic \( B_z \) algorithm is an iterative method to find the conductivity \( \sigma \) satisfying (2). However, the harmonic \( B_z \) algorithm may fail to reconstruct the conductivity in the human experiments, since there exist local regions where are defected by MR signal void. SNR (signal to noise ratio) of \( B_z \) data is inverse proportional to MR signal strength \([6]\). Therefore, the reconstruction result can be entirely corrupted by the noise coming from the defected local region. Moreover, if there exists extremely low conductivity region in the body, the current path way may be almost parallel along the region’s boundaries. It makes the matrix consisting by current density in (2) singular. Hence, we may not guarantee the conductivity reconstruction without the artifact coming from the inversion of the matrix. For those reason, it will be difficult to continue iterative step to find the conductivity \( \sigma \).

Therefore, it has demanded a kind of non-iterative conductivity reconstruction algorithm. In this talk, we briefly review two previous studies about non-iterative conductivity reconstruction \([12,13]\). We discuss about the possibility of reconstruction of the absolute conductivity with high contrast based on non-iterative method. Also, we present several numerical examples for helping the understanding.

REFERENCES


