From Compressed Sensing to Deep Learning: A Journey into the Modern World of Inverse Problems

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ABSTRACT

In this talk I will review a recent journey into the modern world of inverse problems over the past two decades, from compressed sensing to deep learning.

During postdoc years at UIUC, my supervisor Prof. Yoram Bresler proposed a breakthrough idea to overcome Nyquist sampling limits by using sparsity as a regularization term. Inspired by this, I developed the first algorithm for image reconstruction from sparse Fourier samples using sparsity [1]. The experience with this particular topic naturally led me to become actively involved in Compressed Sensing (CS) research for inverse problems when the topic arose as a main research theme in later years.

In fact, one of the reasons I had originally contacted Prof. Yoram Bresler for my postdoctoral position was that he had made a significant contribution to array signal processing, perhaps the most popular signal processing topic of the 1990s. Array signal processing was originally developed for radar devices to leverage multiple measurements measured by an array of sensors, and Prof. Bresler developed one of the most powerful algorithms related to Prony's approach. When I started actively researching CS theory at KAIST, the contemporary CS algorithm for multiple measurements did not exploit the subspace of multiple measurement vectors (MMV), which was in stark contrast to the array signal processing approach. In the pursuit of the missing link between CS and array signal processing, we discovered the Spark reduction principle that can generalize CS theory to the MMV problem [2].

Later, Prof. Emmanuel Candes at Stanford generalized CS theory to the low-rank matrix completion problem [3]. This has inspired the MR imaging experts to exploit the low rank matrix competition for compressed sampling MRI (CS-MRI) and found that imposing the low rank in the Hakel structure matrix of the k-space data can interpolate the missing k-space data. Intrigued by these discoveries, we discovered that completing the low-rank Hankel matrix in the image domain resulted in superior inpainting performance [4]. The result looked like a dual for the low-level Hankel matrix approach to CS-MRI, and we were interested in finding the mathematical origin of the low-rank Hankel matrix in k-space and in the image domain. This led to the discovery of the annihilating filter-based low-rank Hankel matrix (ALOHA) algorithm, which generalized the Prony's method to transform the CS problems into dual-domain data interpolation by a low-rank Hankel matrix [5,6].

In 2016, I came across a paper on convolution framelets by Ingrid Daubechies [7]. In the work, the authors beautifully derived the convolution framelet structure from the decomposition of the Hankel matrix. I distinctly thought there might be a connection to the convolution neural network (CNN). In 2017, we finally realized that the column and row space decomposition of the Hankel matrix can lead to the pooling or convolution filter structure, and the interplay between the local and non-local basis in terms of the convolution

framelet explains the power of CNN. The resulting interpretation of the Deep Convolutional Framelets, published in the SIAM Journal of Imaging Science (SIAM-IS), had a significant impact on the community and has become one of the top 10 most cited papers in SIAM-IS [8].

Early last year, my former student from MILA in Canada offered a seminar on his recent work on diffusion models. Actually, this was the first time I heard about the diffusion models and I didn't understand why diffusion models and score matching are important. At that time, we were investigating how to extend his previous work on self-supervised image denoising based on Stein Unbiased Risk Estimate (SURE). We soon realized that the Stein formula might be the key to self-supervised image denoising and that there might be a connection to score matching. This investigation led us to the discovery of the Noise2Score in NeurIPS 2021 [9] as a general framework for self-supervised image denoising that can unify SURE, Noise2Void, Noise2Self, etc. Furthermore, this naturally leads us to realize the importance of score matching and diffusion models, which is one of our main research topics nowadays.

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