

The ionic Poisson equation for L^p density and application to the Vlasov–Poisson–BGK equation

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ABSTRACT

We analyze the so-called ionic Poisson equation $-\Delta\Phi = \rho - e^\Phi$ on the periodic domain \mathbb{T}^n and show that it admits a unique weak solution $\Phi \in H^1(\mathbb{T}^n)$ whenever $\rho \in L^p$ with $p > 1$. As an application we establish global-in-time existence of weak solutions to the ionic Vlasov–Poisson equations, subject to a BGK-type operator. We also perform careful analysis to verify that a kinetic entropy inequality holds for the solutions we construct. Our motivation for this work is due to that this entropy inequality implies the ionic Vlasov–Poisson–BGK equations converge in the hydrodynamic limit to the ionic Euler–Poisson equations with isentropic pressure. This limiting scheme is discussed rigorously in forthcoming work.

REFERENCES

1. Iacobelli, M. and Griffin-Pickering, M., “Global well-posedness for the Vlasov-Poisson system with massless electrons in the 3-dimensional torus”, *Communications in Partial Differential Equations*, Vol. 46:10, 2021, pp. 1892-1939.
2. Koo, D. and Song, S., “Global mild solutions to a BGK model for barotropic gas dynamics”, arXiv:2409.04129, 2024.